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CP Violation in Neutrino Oscillation Due to Planck Scale Effects

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Abstract We consider non renormalization $1/M_x$ interaction term as a perturbation of the neutrino mass matrix. We find that for the degenerate neutrino mass spectrum. We assume that the neutrino masses and mixing arise through physics at a scale intermediate between Planck Scale and the electroweak scale. We also assume, above the electroweak breaking scale, neutrino masses are nearly degenerate and their mixing is bimaximal. The perturbation generates a non zero value of θ_{13}, which is within reach of the high performance neutrino factory. In this paper, we find that the non zero value of θ_{13} due to Planck scale effects indicates the possibility of CP violation.

Keywords CP violation · Neutrino oscillation

1 Introduction

The origin of CP violation is still mystery in particle physics. Recent advance in neutrino physics observation mainly of astrophysical observation suggested the existence if tiny neutrino mass. Neutrino are massive and there is mixing in lepton sector, this indicates to imagine that there occurs CP violation effects in lepton sector. Several physicist have considered whether, we can see CP violation effects in lepton sector through long baseline neutrino oscillation experiments. The neutrino oscillation probability, in general depends on six parameter two independent mass square difference Δ_{21} and Δ_{31}, three mixing angles θ_{12}, θ_{23}, θ_{13} and one CP violating phase δ. Significance of CP, T violation for neutrinos have been given by [1, 2]. CP violation arise as three or more generation [3, 4]. CP violation in neutrino oscillation is interesting because it relates directly to CP phase parameter in the mixing for $n > 3$ degenerate neutrino. The experimental study of neutrino oscillations has indicates by existing non vanish neutrino masses and neutrino mixing. The most important tasks for experiment, precise determination of the measured values that are already known

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and the measurement of the mixing angle θ_{13}, where at present is only limited by an upper bound, the determination of the sign of $\Delta_{31} = m_3^2 - m_1^2$ and the CP violating phase δ. All CP violating quantities in oscillation phenomenon are determine in term of Jarlskog Determinant J, which is the standard notation for mixing angle is given by

$$J = \frac{1}{8}\sin 2\theta_{12}\sin 2\theta_{23}\sin 2\theta_{13}\cos\theta_{13}\sin\delta. \quad (1)$$

We can write down the compact formula for the difference of transition probability between conjugate channel.

$$\Delta P(\alpha,\beta) = P(\nu_\mu \to \nu_e) - P(\bar{\nu_\mu} \to \bar{\nu_e}), \quad (2)$$

where

$$(\alpha,\beta) = (e,\mu), (\mu,\tau), (\tau,e).$$

The main physical goal in future experiment are the determination of the unknown parameter θ_{13} and upper bound $\sin^2 2\theta_{13} < 0.01$ is obtained for the ref [5]. In particular, the observation of δ is quites interesting for the point of view that δ related to the origin of the matter in the universe. The determination of δ is the final goal of the future experiments. A lot of experiment discuss the strategies for measured δ in long baseline neutrino experiments with super-beam, β-beam and neutrino factories. CP violation in neutrino oscillation are given in Sect. 2. In Sect. 3 give the neutrino oscillation parameter and possible CP violation due to Planck scale effects. In Sect. 4 give the numerical results and finally give the conclusions.

2 CP Violation in Neutrino Oscillation

CP violation effects in neutrino oscillations, we find that a comparison of $\nu_\alpha \to \nu_\beta$ and $\overline{\nu}_\alpha \to \overline{\nu}_\beta$ oscillation probability at neutrino factories would give precession test of CP violation. In neutrino oscillation CP is not conserved

If

$$P_{\alpha\beta}(L) \neq P_{\overline{\alpha}\overline{\beta}}(L), \qquad \beta \neq \alpha, \quad (3)$$

where $P_{\alpha\beta}(L)$ is $\nu_\alpha \to \nu_\beta$ oscillation probabilities, L is Baseline length.

The effective CPT violation interaction for neutrino is of the form [6–8]

$$\overline{\nu}_{L^\alpha} b^\mu_{\alpha\beta}\gamma_\mu \nu^\beta_L, \quad (4)$$

where α and β are flavor indices.

In presence of this CPT violation, the neutrino energy acquires an additional term which comes from the matrix $b_{\alpha\beta}$. For anti-neutrino, this term have the opposite sign. The energy eigen value of neutrino (in ultra relativistic case) are obtained by diagonalizing the Herniation Matrix given by

$$\frac{m^2}{2p} + b, \quad (5)$$

where $m^2 = mm^\dagger$ is the Herniation mass square matrix.

Solution to solar and atmospheric neutrino problems leads to $\Delta_{\text{solar}} \ll \Delta_{\text{atmos}}$. We choose mass eigen state such that $\Delta_{\text{solar}} = \Delta_{21}$ and $\Delta_{\text{atmos}} = \Delta_{31}$. Neutrino oscillation experiments gives

$$\Delta_{21} \ll \Delta_{31} \simeq \Delta_{32}.$$

For algebraic simplicity, the probability of ν_μ oscillate to ν_e simplified as

$$P_{\mu e} = \sin^2\theta_{23}\sin^2 2\theta_{13}\sin^2\left(\frac{\Delta_{31}L}{4E}\right). \tag{6}$$

The expression for neutrino oscillation probability satisfy a conventional formula [8], when $\Delta_{21} \leqslant \Delta_{31}$

$$P_{\mu e} = \sin^2\theta_{23}\sin^2 2\theta_{13}\sin^2\left(\frac{\Delta_{31}L}{4E} + \frac{\delta b L}{2}\right). \tag{7}$$

$\Delta_{31} = m_3^2 - m_1^2$ is in eV^2 unit. The δb has unit of energy (GeV). For anti-neutrino sign of δb is opposite. We assume equal mass of neutrino and anti-neutrino. For simplicity we have assumed that the two mixing angles that diagonalize the matrix m^2 and b are equal. The expression for anti-neutrino oscillation probability

$$P_{\overline{\mu e}} = \sin^2\theta_{23}\sin^2 2\theta_{13}\sin^2\left(\frac{\Delta_{31}L}{4E} - \frac{\delta b L}{2}\right). \tag{8}$$

The difference between $P_{\mu e}$ and $P_{\overline{\mu e}}$ is given by

$$\Delta P_{\mu e}^{CP} = P_{\mu e} - P_{\overline{\mu e}} = \sin^2\theta_{23}\sin^2 2\theta_{13}\sin(\delta b L)\sin^2\left(\frac{\Delta_{31}L}{2E}\right), \tag{9}$$

can be used to test for CP violation. Above dispersion relation of $\Delta P_{\mu e}^{\text{CP}}$ shows that, CP violation depending on which term is larger for a given set of parameter. The presence of CPT violation [6] can be detected provide $\delta b > 3 \times 10^{-23}$ GeV.

3 Neutrino Oscillation Parameter and Possible CP Violation Due to Planck Scale Effects

The neutrino mass matrix is assumed to be generated by the see saw mechanism [10–12]. We assume that the dominant part of neutrino mass matrix arise due to GUT scale operators and the lead to bi-maximal mixing. The effective gravitational interaction of neutrino with Higgs field can be expressed as $SU(2)_L \times U(1)$ invariant dimension-5 operator [9],

$$L_{\text{grav}} = \frac{\lambda_{\alpha\beta}}{M_{pl}}(\psi_{A\alpha}\epsilon\psi_C)C_{ab}^{-1}(\psi_{B\beta}\epsilon_{BD}\psi_D) + h.c. \tag{10}$$

Here and every where we use Greek indices α, β for the flavour states and Latin indices i, j, k for the mass states. In the above equation $\psi_\alpha = (\nu_\alpha, l_\alpha)$ is the Lepton doublet, $\phi = (\phi^+, \phi^\circ)$ is the Higgs doublet and $M_{pl} = 1.2 \times 10^{19}$ GeV is the Planck mass λ is a 3×3 matrix in a flavour space with each elements $O(1)$. The Lorentz indices $a, b = 1, 2, 3, 4$ are contracted with the charge conjugation matrix C and the $SU(2)_L$ isospin indices $A, B, C, D = 1, 2$ are contracted with $\epsilon = i\sigma_2$, σ_m $(m = 1, 2, 3)$ are the Pauli matrices. After spontaneous electroweak symmetry breaking the Lagrangian in (10) generated additional term of neutrino mass matrix

$$L_{\text{mass}} = \frac{v^2}{M_{pl}} \lambda_{\alpha\beta} \nu_\alpha C^{-1} \nu_\beta, \tag{11}$$

where $v = 174$ GeV is the *VEV* of electroweak symmetric breaking. We assume that the gravitational interaction is "flavour blind" that is $\lambda_{\alpha\beta}$ is independent of α, β indices. Thus the Planck scale contribution to the neutrino mass matrix is

$$\mu\lambda = \mu \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}, \tag{12}$$

where the scale μ is

$$\mu = \frac{v^2}{M_{pl}} = 2.5 \times 10^{-6} \text{ eV}. \tag{13}$$

We take (12) as perturbation to the main part of the neutrino mass matrix, that is generated by GUT dynamics. To calculate the effects of perturbation on neutrino observables. The calculation developed in an earlier paper [9]. A natural assumption is that unperturbed (0th order mass matrix) M is given by

$$\mathbf{M} = U^* \operatorname{diag}(M_i) U^\dagger, \tag{14}$$

where, $U_{\alpha i}$ is the usual mixing matrix and M_i, the neutrino masses is generated by Grand unified theory. Most of the parameter related to neutrino oscillation are known, the major expectation is given by the mixing elements U_{e3}. We adopt the usual parametrization.

$$\frac{|U_{e2}|}{|U_{e1}|} = \tan\theta_{12}, \tag{15}$$

$$\frac{|U_{\mu 3}|}{|U_{\tau 3}|} = \tan\theta_{23}, \tag{16}$$

$$|U_{e3}| = \sin\theta_{13}. \tag{17}$$

In term of the above mixing angles, the mixing matrix is

$$U = \operatorname{diag}(e^{if1}, e^{if2}, e^{if3}) R(\theta_{23}) \Delta R(\theta_{13}) \Delta^* R(\theta_{12}) \operatorname{diag}(e^{ia1}, e^{ia2}, 1). \tag{18}$$

The matrix $\Delta = \operatorname{diag}(e^{\frac{1\delta}{2}}, 1, e^{\frac{-i\delta}{2}})$ contains the Dirac phase. This leads to CP violation in neutrino oscillation $a1$ and $a2$ are the so called Majoring phase, which effects the neutrino less double beta decay. $f1$, $f2$ and $f3$ are usually absorbed as a part of the definition of the charge lepton field. Planck scale effects will add other contribution to the mass matrix that gives the new mixing matrix can be written as [9]

$$U' = U(1 + i\delta\theta),$$

$$\begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}$$
$$+ i \begin{pmatrix} U_{e2}\delta\theta_{12}^* + U_{e3}\delta\theta_{23,}^* & U_{e1}\delta\theta_{12} + U_{e3}\delta\theta_{23}^*, & U_{e1}\delta\theta_{13} + U_{e3}\delta\theta_{23}^* \\ U_{\mu 2}\delta\theta_{12}^* + U_{\mu 3}\delta\theta_{23,}^* & U_{\mu 1}\delta\theta_{12} + U_{\mu 3}\delta\theta_{23}^*, & U_{\mu 1}\delta\theta_{13} + U_{\mu 3}\delta\theta_{23}^* \\ U_{\tau 2}\delta\theta_{12}^* + U_{\tau 3}\delta\theta_{23}^*, & U_{\tau 1}\delta\theta_{12} + U_{\tau 3}\delta\theta_{23}^*, & U_{\tau 1}\delta\theta_{13} + U_{\tau 3}\delta\theta_{23}^* \end{pmatrix}. \tag{19}$$

Where $\delta\theta$ is a Hermitian matrix that is first order in μ [9, 13]. The first order mass square difference $\Delta M_{ij}^2 = M_i^2 - M_j^2$, get modified [9, 13] as

$$\Delta M_{ij}'^2 = \Delta M_{ij}^2 + 2(M_i \mathrm{Re}(m_{ii}) - M_j \mathrm{Re}(m_{jj}), \tag{20}$$

where

$$m = \mu U^t \lambda U,$$
$$\mu == \frac{v^2}{M_{pl}} = 2.5 \times 10^{-6} \text{ eV}.$$

The change in the elements of the mixing matrix, which we parametrized by $\delta\theta$ [9], is given by

$$\delta\theta_{ij} = \frac{i\mathrm{Re}(m_{jj})(M_i + M_j) - \mathrm{Im}(m_{jj})(M_i - M_j)}{\Delta M_{ij}'^2}. \tag{21}$$

The above equation determine only the off diagonal elements of matrix $\delta\theta_{ij}$. The diagonal element of $\delta\theta_{ij}$ can be set to zero by phase invariance. Using (19), we can calculate neutrino mixing angle due to Planck scale effects,

$$\frac{|U'_{e2}|}{|U'_{e1}|} = \tan\theta'_{12}, \tag{22}$$

$$\frac{|U'_{\mu 3}|}{|U'_{\tau 3}|} = \tan\theta'_{23}, \tag{23}$$

$$|U'_{e3}| = \sin\theta'_{13}. \tag{24}$$

For degenerate neutrinos, $M_3 - M_1 \cong M_3 - M_2 \gg M_2 - M_1$, because $\Delta_{31} \cong \Delta_{32} \gg \Delta_{21}$. Thus, from the above set of equations, we see that U'_{e1} and U'_{e2} are much larger than U'_{e3}, $U'_{\mu 3}$ and $U'_{\tau 3}$. Hence we can expect much larger change in θ_{12} compared to θ_{13} and θ_{23} [17]. As one can see from the above expression of mixing angle due to Planck scale effects, depends on new contribution of mixing $U' = U(1 + i\delta\theta)$.

The possible CP violation due to new contribution of mixing (Planck scale effects) is given by

$$\Delta P_{\mu e}'^{\mathrm{CP}} = P'_{\mu e} - P'_{\overline{\mu e}} = \sin^2\theta'_{23}\sin^2 2\theta'_{13}\sin(\delta b L)\sin^2\left(\frac{\Delta_{31}L}{2E}\right). \tag{25}$$

Table 1 Modified mixing angles for some sample of a_1 and a_2. Input value are $\Delta_{31} = 0.002\ \text{eV}^2$, $\theta_{12} = 45°$, $\theta_{23} = 45°$, $\theta_{13} = 0°$

$a1$	$a2$	Modified Mixing Angle θ'_{13}
0	0	0.28
0	45	0.22
0	90	0.13
0	135	0.22
0	180	0.28
45	0	0.22
45	45	0.20
45	90	0.09
45	135	0.14
45	180	0.22
90	0	0.14
90	45	0.10
90	90	0.00005
90	135	0.10
90	180	0.14
135	0	0.22
135	45	0.14
135	90	0.09
135	135	0.20
135	180	0.22
180	0	0.22
180	45	0.13
180	90	0.22
180	135	0.22
180	180	0.28

4 Results and Discussions

We expect the mixing angles from GUT scale operators to be determine by some symmetry. We assume that, just above the electroweak breaking scale, the neutrino masses are nearly degenerate and the mixing are bimaximal, with the value of the mixing angle as $\theta_{12} = \pi/4$, $\theta_{23} = \pi/4$ and $\theta_{13} = 0$. Taking the common degenerate neutrino mass to be 2 eV, which is the upper limit coming from tritium beta decay [14]. We compute the modified mixing angles using (15)–(17). We have taken $\Delta_{31} = 0.002\ \text{eV}^2$ [15] and $\Delta_{21} = 0.00008\ \text{eV}^2$ [16]. For simplicity we have set the charge lepton phases $f_1 = f_2 = f_3 = 0$. Since we have set the $\theta_{13} = 0$, the Dirac phase δ drops out of the zeroth order mixing angle. In Table 1, we list the modified neutrino mixing angles for some sample value of $a1$ and $a2$. Due to Planck scale effects, only θ_{12} have resonable deviation and θ_{23}, θ_{13} deviation is very small less then $0.3°$ [17]. From Table 1, we can see for bimaximal mixing pattern $\theta_{12} = \pi/4$, $\theta_{23} = \pi/4$, and $\theta_{13} = 0$. We get the non zero value of modified mixing angle $\theta'_{13} < 0.3°$ and possible CP violation parameter $\Delta P'^{CP}_{\mu e} \neq 0$ gives non zero value. If $\theta_{13} \neq 0$ due to Planck scale effects, we can extracted the possible CP violation Dirac phase

$$|\delta| = \sin^{-1}\left(\left|\frac{8\text{Im}(U'_{e1}U'_{e2}U'_{\mu 1}U'_{\mu 2})}{\cos\theta'_{13}\sin 2\theta'_{12}\sin 2\theta'_{23}\sin 2\theta'_{13}}\right|\right), \tag{26}$$

Due to Planck scale effects, non zero value of mixing angle θ_{13} indicate the possible CP violation in neutrino sector.

5 Conclusions

We assume that the main part of neutrino masses and mixing from GUT scale operator. We considered these to be 0th order quanties. We further assume that GUT scale symmetry constrain the neutrino mixing angles to be bimaximal. The gravitational interaction of lepton field with S.M. Higgs field give rise to a $SU(2)_L \times U(1)$ invariant dimension-5 effective Lagrangian give originally by Weinberg [18]. On electroweak symmetry breaking this operators leads to additional mass terms. We considered these to be perturbation of GUT scale mass terms. We compute the first order correction to neutrino mass eigen value and mixing angles. In this paper, we have discuss of the CP violation in the neutrino sector. We have presented a simple CP violation term $\Delta P_{\mu e}^{CP}$, which is independent of CP phase. Bimaximal scenario of neutrino mixing due to Planck scale effects [17] gives the range of mixing angle. The range of mixing angle θ_{13} is given by [12] is $\theta_{13} = (0.004\text{–}0.3^\circ)$. In this paper, we studied how Planck scale can probe a signal of CP violation. We find the change in the mixing angle due to bimaximal mixing. We get the non zero value of $\theta_{13} = (0.10^\circ\text{–}0.28^\circ)$. This indicate the there is possibility of CP violation due to Planck scale effects. In this paper, finally we wish to made a important comment. Due to Planck scale effects mixing angle $\theta_{13} \neq 0$ indicates the possibility of CP violation.

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